Substituting Eqs. (2) and (4) into Eq. (1a), we can obtain

$$
\begin{aligned}
D_1 &= \left(e_{11} - e_{13}c_{13}^E/c_{33}^E\right)S_1 + \left(e_{12} - e_{13}c_{23}^E/c_{33}^E\right)S_2 \\
&\quad + \left(e_{16} - e_{13}c_{36}^E/c_{33}^E\right)S_6 + \left[\varepsilon_{11}^S + (e_{13})^2/c_{33}^E\right]E_1 \\
&\quad + \left(\varepsilon_{12}^S + e_{13}e_{23}/c_{33}^E\right)E_2 + \left(\varepsilon_{13}^S + e_{13}e_{33}/c_{33}^E\right)E_3 \\
D_2 &= \left(e_{21} - e_{23}c_{13}^E/c_{33}^E\right)S_1 + \left(e_{22} - e_{23}c_{23}^E/c_{33}^E\right)S_2 \\
&\quad + \left(e_{26} - e_{23}c_{36}^E/c_{33}^E\right)S_6 + \left(\varepsilon_{12}^S + e_{13}e_{23}/c_{33}^E\right)E_1 \\
&\quad + \left[\varepsilon_{22}^S + (e_{23})^2/c_{33}^E\right]E_2 + \left(\varepsilon_{23}^S + e_{23}e_{33}/c_{33}^E\right)E_3 \\
D_3 &= \left(e_{31} - e_{33}c_{13}^E/c_{33}^E\right)S_1 + \left(e_{32} - e_{33}c_{23}^E/c_{33}^E\right)S_2 \\
&\quad + \left(e_{36} - e_{33}c_{36}^E/c_{33}^E\right)S_6 + \left(\varepsilon_{13}^S + e_{13}e_{33}/c_{33}^E\right)E_1 \\
&\quad + \left(\varepsilon_{23}^S + e_{23}e_{33}/c_{33}^E\right)E_2 + \left[\varepsilon_{33}^S + (e_{33})^2/c_{33}^E\right]E_3
\end{aligned}
\tag{6}
$$

Equations (5) and (6) can be written as the following forms that are the plane form piezoelectric constitutive equations:

$$
\begin{bmatrix} T_1 \\ T_2 \\ T_6 \end{bmatrix} = \begin{bmatrix} c_{11}^X & c_{12}^X & c_{16}^X \\ c_{12}^X & c_{22}^X & c_{26}^X \\ c_{16}^X & c_{26}^X & c_{66}^X \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_6 \end{bmatrix} - \begin{bmatrix} e_{11}^X & e_{21}^X & e_{31}^X \\ e_{12}^X & e_{22}^X & e_{32}^X \\ e_{16}^X & e_{26}^X & e_{36}^X \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{7a}
$$

$$
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} e_{11}^X & e_{12}^X & e_{16}^X \\ e_{21}^X & e_{22}^X & e_{26}^X \\ e_{31}^X & e_{32}^X & e_{36}^X \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_6 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11}^X & \varepsilon_{12}^X & \varepsilon_{13}^X \\ \varepsilon_{12}^X & \varepsilon_{22}^X & \varepsilon_{23}^X \\ \varepsilon_{13}^X & \varepsilon_{23}^X & \varepsilon_{33}^X \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{7b}
$$

where

$$
c_{ij}^X = c_{ij}^E - c_{i3}^E c_{j3}^E / c_{33}^E, \qquad e_{kj}^X = e_{kj} - e_{k3}c_{j3}^E / c_{33}^E
$$
$$
\varepsilon_{kl}^X = \varepsilon_{kl}^S + e_{k3}e_{l3}/c_{33}^E \tag{8}
$$

are the corresponding new elastic stiffness matrix, piezoelectric stress/charge matrix, and permittivity matrix, respectively; i, $j = 1$, 2, 6 and k, $l = 1$, 2, 3, and the superscript X denotes the constants in the new equations.

To date, the piezoelectric materials of a commonly discussed and used sensor/actuator are polyvinylidene fluoride polymer (PVDF) or PZT (lead, zirconate, titanate), etc., which are at least orthotropic. The pole direction of such a sensor/actuator is in its thickness direction. In addition, the electric field in the sensor/actuator can be treated as a uniform electric field. Then among the components of the electric field and electric displacement the possible nonzero components are E_3 and D_3. For later uses, according to Eqs. (7) and (8) and their concrete material constants, we write the piezoelectric constitutive equations for an orthotropic plate shape sensor/actuator:

$$
\begin{bmatrix} T_1 \\ T_2 \\ T_6 \end{bmatrix} = [c] \begin{bmatrix} S_1 \\ S_2 \\ S_6 \end{bmatrix} - E_3 \begin{bmatrix} e_{31} - \left(c_{13}^E/c_{33}^E\right)e_{33} \\ e_{32} - \left(c_{23}^E/c_{33}^E\right)e_{33} \\ 0 \end{bmatrix} \tag{9a}
$$

$$
D_3 = e_{31}^X S_1 + e_{32}^X S_2 + e_{36}^X S_6 + \varepsilon_{33}^X E_3 \tag{9b}
$$

where

$$
[c] = \begin{bmatrix} c_{11}^E - \left(c_{13}^E\right)^2/c_{33}^E & c_{12}^E - c_{13}^E c_{23}^E/c_{33}^E & 0 \\ c_{12}^E - c_{13}^E c_{23}^E/c_{33}^E & c_{22}^E - \left(c_{23}^E\right)^2/c_{33}^E & 0 \\ 0 & 0 & c_{66}^E \end{bmatrix}
$$
$$
e_{31}^X = e_{31} - \left(c_{13}^E/c_{33}^E\right)e_{33}, \qquad e_{32}^X = e_{32} - \left(c_{23}^E/c_{33}^E\right)e_{33} \tag{10}
$$
$$
\varepsilon_{33}^X = \varepsilon_{33}^S + \left(e_{33}^2/c_{33}^E\right)
$$

Conclusions

We have given the exact formulas of the piezoelectric constitutive equations for a plate shape sensor/actuator, which are complements of the theory of laminated piezoelectric plates. According to our formulas, the constants in the new equations can be obtained directly from those in the general piezoelectric constitutive equations, whereas the latter are given by the manufacturer or can be directly calculated from the manufacturer-given constants according to the existing knowledge in piezoelectricity. The other three forms of the plane form piezoelectric constitutive equations corresponding to those of the general piezoelectric constitutive equations can be obtained from our equations through simple algebraic calculations. Our results can be used when one analyzes the motion of a laminated piezoelectric plate.

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Dispersion of Axisymmetric Elastic Waves in Thick-Walled Orthotropic Pipes

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Introduction

A METHOD for studying harmonic wave propagation in thick orthotropic cylinders of infinite length was presented by Markuš and Mead.[1] Displacements were modeled by trigonometric functions and Frobenius power series. Harmonic wave propagation in shells and rods of infinite length was studied by Mazúch.[2] Relations for a finite element model were derived for general anisotropy of a linearly elastic material. The generalized eigenvalue problem was solved by the Lanczos method with simple orthogonalization by Mazúch.[3] Jing and Tzeng[4] presented an approximate elasticity solution for arbitrarily laminated anisotropic cylindrical closed shells of finite length with simply supported ends.

In this Note, a modification of the last method is used to obtain dispersion curves for the axisymmetric problem of arbitrarily laminated, orthotropic, unclosed, cylindrical pipes of infinite length. The first five dispersion curves are obtained. A convergence study is performed for a single orthotropic layer with the elastic moduli

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published by Pagano.[5] Numerical computations were carried out by means of a computer program based on Wolfram's software.[6]

Problem Description

Consider an infinitely long pipe in a cylindrical coordinate system with the radial, circumferential, and axial coordinates r, θ, and x, respectively. Imagine the whole layer, with the inner, middle, and outer radii R_I, R_m, and R_O, respectively, divided into M thin coaxial laminae. The R_k is the middle radius of the kth lamina. The generalized Hooke's law equations (constitutive equations) for each lamina are given by

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_r \\ \tau_{r\theta} \\ \tau_{xr} \\ \tau_{x\theta} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{r\theta} \\ \gamma_{xr} \\ \gamma_{x\theta} \end{bmatrix} \quad (1)$$

The C_{ij} expressed in Eqs. (1) are nonzero elastic moduli. The index k of the kth lamina is omitted for simplicity. The strain-displacement relations are expressed as follows:

$$\varepsilon_x = \frac{\partial u_x}{\partial x}, \qquad \varepsilon_\theta = \frac{u_r}{r}, \qquad \varepsilon_r = \frac{\partial u_r}{\partial r}, \qquad \gamma_{xr} = \frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial x} \quad (2)$$

Because the axisymmetric problem is considered, $\gamma_{r\theta}$, $\gamma_{x\theta}$, and $u_\theta = 0$. The differential equations of motion are written as

$$\frac{1}{r}\frac{\partial(r\sigma_r)}{\partial r} + \frac{\partial \tau_{xr}}{\partial x} - \frac{\sigma_\theta}{r} = \rho\frac{\partial^2 u_r}{\partial t^2}$$
$$\frac{1}{r}\frac{\partial(r\tau_{xr})}{\partial r} + \frac{\partial \sigma_x}{\partial x} = \rho\frac{\partial^2 u_x}{\partial t^2} \quad (3)$$

where ρ is the density of the material and t is time. After substituting Eqs. (1) and (2) into Eqs. (3), the governing equations in terms of displacements for each lamina become

$$C_{33}\frac{\partial^2 u_r}{\partial r^2} + C_{33}\frac{\partial u_r}{r\partial r} + C_{55}\frac{\partial^2 u_r}{\partial x^2} - C_{22}\frac{u_r}{r^2} + (C_{13} - C_{12})\frac{\partial u_x}{r\partial x} + (C_{13} + C_{55})\frac{\partial^2 u_x}{\partial x\partial r} = \rho\frac{\partial^2 u_r}{\partial t^2} \quad (4a)$$

$$(C_{12} + C_{55})\frac{\partial u_r}{r\partial x} + (C_{13} + C_{55})\frac{\partial^2 u_r}{\partial r\partial x} + C_{11}\frac{\partial^2 u_x}{\partial x^2} + C_{55}\frac{\partial^2 u_x}{\partial r^2} + C_{55}\frac{\partial u_x}{r\partial r} = \rho\frac{\partial^2 u_x}{\partial t^2} \quad (4b)$$

Equations (4) are coupled partial differential equations with variable coefficients. It is not possible to solve them in a closed form. An approximation is made for simplifying these equations. Introducing the radial local coordinate $\xi_k = r - R_k$ located at the center of the kth lamina and making the approximation $\xi_k/R_k \ll 1$, the following equations for the variable parts of the coefficients in Eqs. (3) are assumed:

$$1/r = (1/R_k)(1 - \eta_k), \qquad 1/r^2 = \left(1\big/R_k^2\right)(1 - 2\eta_k) \quad (5)$$

where $\eta_k = \xi_k/R_k$. The dimensionless coordinate η_k can take on any value in the interval $\langle \eta_k^l, \eta_k^u \rangle = \langle -h_k/R_k, h_k/R_k \rangle$, where h_k is the half-thickness of the kth lamina and the superscripts l and u denote the lower and upper limits of η_k, respectively.

Solution of Governing Equations

To determine dispersion relations, it is assumed that steady-state harmonic waves propagate along the x axis of the pipe. For the axisymmetric problem, a solution for the kth lamina in time t at a point x takes the form

$$u_{rk} = U_k \cos[K(x - ct)], \qquad u_{xk} = W_k \sin[K(x - ct)] \quad (6)$$

where $K = 2\pi/\lambda$ is the wave number, c is the phase velocity of the elastic wave with the wavelength λ, and U_k and W_k are unknown amplitudes as functions of the dimensionless radial coordinate η_k.

Inserting Eqs. (5) and (6) into Eqs. (4), with respect to $\eta_k \ll 1$, yields

$$\left(C_{22} + C_{55}K^2R_k^2\right)U_k + (C_{12} - C_{13})KR_kW_k - C_{33}U_k' - (C_{13} + C_{55})KR_kW_k' - C_{33}U_k'' = c^2\rho K^2R_k^2U_k \quad (7a)$$

$$(C_{12} + C_{55})KR_kU_k + C_{11}K^2R_k^2W_k + (C_{13} + C_{55})KR_kU_k' - C_{55}W_k' - C_{55}W_k'' = c^2\rho K^2R_k^2W_k \quad (7b)$$

The prime denotes the derivative with respect to η_k. Equations (7) are coupled ordinary differential equations with constant coefficients for the amplitudes U_k and W_k. The general solution of Eqs. (7) for the kth artificial lamina is sought in the form

$$U_k = \sum_{i=1}^{4} A_{ki}\exp(\alpha_{ki}\eta_k), \qquad W_k = \sum_{i=1}^{4} A_{ki}\Psi_{ki}\exp(\alpha_{ki}\eta_k) \quad (8)$$

The $4M$ constants α_{ki}, $k = 1, 2, \ldots, M$, $i = 1, 2, \ldots, 4$, are obtained for each k by solving the characteristic equation

$$a_4\alpha_{ki}^4 + a_3\alpha_{ki}^3 + a_2\alpha_{ki}^2 + a_1\alpha_{ki} + a_0 = 0 \quad (9)$$

obtained by substituting the amplitudes from Eqs. (8) into Eqs. (7). The $4M$ constants Ψ_{ki} are roots of Eq. (7a) after including Eqs. (8). The $4M$ coefficients A_{ki} in Eqs. (8) can be determined by satisfying the boundary conditions on the inner and outer surfaces of the pipe and the continuity conditions for stresses and displacements on the boundaries between the coincident laminae. The amplitudes of the stresses σ_r and τ_{xr}, e.g., $\Sigma_{r,k}$ and $T_{xr,k}$, appear in both kinds of conditions:

$$\Sigma_{r,k}(R_k, \eta_k) = \frac{C_{23}U_k(\eta_k)}{R_k} + C_{13}KW_k(\eta_k) + \frac{C_{33}U_k'(\eta_k)}{R_k} \quad (10a)$$

$$T_{xr,k}(R_k, \eta_k) = -C_{55}KU_k(\eta_k) + \frac{C_{55}W_k'(\eta_k)}{R_k} \quad (10b)$$

expressed from Eqs. (1) by using the same procedure as for obtaining Eqs. (7). The boundary and continuity conditions create a system of $4M$ homogeneous linear algebraic equations for the coefficients A_{ki} in Eqs. (8):

$$\Sigma_{r,1}\left(R_1, \eta_1^l\right) = 0, \qquad T_{xr,1}\left(R_1, \eta_1^l\right) = 0$$
$$U_k\left(\eta_k^u\right) = U_{k+1}\left(\eta_{k+1}^l\right), \qquad W_k\left(\eta_k^u\right) = W_{k+1}\left(\eta_{k+1}^l\right)$$
$$\Sigma_{r,k}\left(R_k, \eta_k^u\right) = \Sigma_{r,k+1}\left(R_{k+1}, \eta_{k+1}^l\right) \quad (11)$$
$$T_{xr,k}\left(R_k, \eta_k^u\right) = T_{xr,k+1}\left(R_{k+1}, \eta_{k+1}^l\right)$$
$$\Sigma_{r,M}\left(R_M, \eta_M^u\right) = 0, \qquad T_{xr,M}\left(R_M, \eta_M^u\right) = 0$$

where $k = 1, 2, \ldots, M - 1$. For obtaining a nontrivial solution of Eqs. (11), the determinant of the system considered is to be zero:

$$|\beta_{ij}| = 0, \qquad i, j = 1, 2, \ldots, 4M \quad (12)$$

Equation (12) is the dispersion equation, and $|\beta_{ij}|$ is the dispersion determinant. The elements of the dispersion determinant are functions of the elastic moduli C_{ij}, radii R_k, wave number K, and phase velocity c.

Numerical Results and Discussion

Consider a cylindrical pipe of unidirectional boron–epoxy composite. The fibers coincide with the coordinate θ. The layer Young's

Table 1 Convergence study for velocity c'

K'	No.	No. of laminae M 1	2	5	10	FEM[a]
0.001	1	1.426	1.425	1.424	1.424	1.424
0.250	1	1.423	1,420	1.419	1.419	1.418
	2	2.296	2.314	2.311	2.311	2.305
	3	3.017	3.048	3.046	3.046	3.007
	4	3.975	3.987	4.002	4.002	4.000
	5	5.365	5.485	5.567	5.578	5.551
0.500	1	1.384	1.362	1.354	1.353	1.344
	2	1.612	1.497	1.501	1.501	1.493
	3	1.799	1.811	1.813	1.813	1.808
	4	2.200	2.222	2.234	2.235	2.231
	5	2.836	2.881	2.917	2.921	2.911
1.000	1	0.901	0.898	0.894	0.893	0.885
	2	1.225	1.255	1.267	1.268	1.263
	3	1.467	1.477	1.483	1.484	1.483
	4	1.492	1.491	1.491	1.491	1.490
	5	1.796	1.801	1.809	1.810	1.809
2.000	1	0.692	0.690	0.689	0.688	0.686
	2	0.816	0.832	0.840	0.841	0.838
	3	1.120	1.122	1.126	1.127	1.125
	4	1.352	1.354	1.355	1.356	1.355
	5	1.461	1.462	1.463	1.463	1.463
3.000	1	0.650	0.648	0.647	0.647	0.646
	2	0.705	0.714	0.720	0.720	0.719
	3	0.892	0.893	0.896	0.896	0.895
	4	1.100	1.102	1.103	1.102	1.102
	5	1.276	1.277	1.278	1.278	1.278

[a]Finite element method.

and shear moduli E and G and Poisson ratios ν are taken as $E_\theta = 30$ Mpsi, $E_r = E_x = 3$ Mpsi, $G_{r\theta} = G_{x\theta} = 1.5$ Mpsi, $G_{xr} = 0.6$ Mpsi, and $\nu_{r\theta} = \nu_{x\theta} = \nu_{xr} = 0.25$. The corresponding elastic moduli are

$$[C_{ij}] = \begin{bmatrix} 3.234 & 1.017 & 0.834 & 0 & 0 & 0 \\ 1.017 & 30.51 & 1.017 & 0 & 0 & 0 \\ 0.834 & 1.017 & 3.234 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5 \end{bmatrix} \text{Mpsi} \qquad (13)$$

The density $\rho = 2000$ kg/m^3. The outer-to-inner-radius ratio $R_O/R_I = 2$. The dispersion equation (12) was solved after introducing the dimensionless wave number K' and the dimensionless phase velocity c':

$$K' = \frac{R_m}{\lambda}, \qquad c' = \frac{c}{\sqrt{C_{44}/\rho}} \qquad (14)$$

where the middle radius of the pipe $R_m = (R_I + R_O)/2$. The denominator of the c' is the phase velocity of the transversal elastic wave propagating in the direction of the axial coordinate x.

Equation (12) was solved as follows. For due values K' from the interval $\langle 0,\ 3 \rangle$, the velocity c' was changed step by step. At each step, the sign of the dispersion determinant was found. If the sign change occurred, c' was determined by the decrease of the step. With respect to computing time, the maximum number of the laminae used was $M = 5$. The lowest five dispersion curves are shown in Fig. 1.

A convergence test of the method was done for chosen wave numbers $K' = 0.001$, 0.250, 0.500, 1.000, 2.000, and 3.000. The results with accuracy to three decimal numbers for the lamina counts $M = 1$, 2, 5, and 10 are in Table 1. For the comparison, the results obtained by Mazúch's approach[2] (finite element method) are listed in the rightmost column of the table. As can be seen, the satisfactory accuracy was already achieved at $M = 5$.

For a given wave number K', there is a different phase velocity c' on each dispersion curve. The distribution of the displacements is the most important for the basic curve. The amplitudes of the radial and axial displacements, normalized to 1 at $r = 1$, are shown in Fig. 2

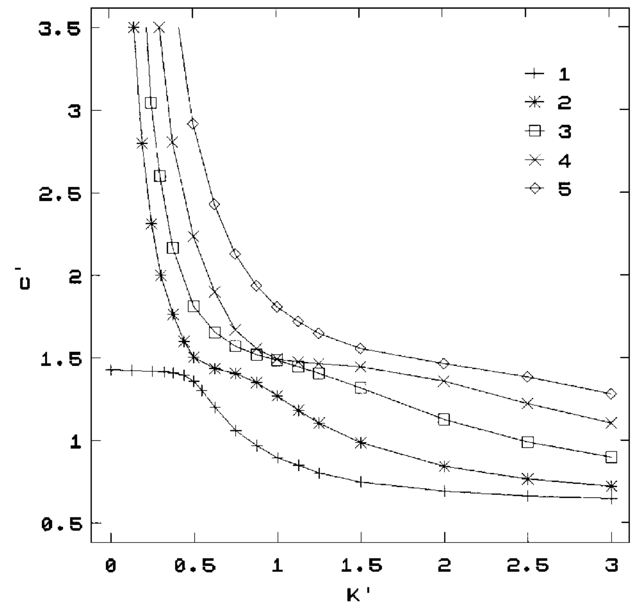

Fig. 1 First five dispersion curves for M = 5.

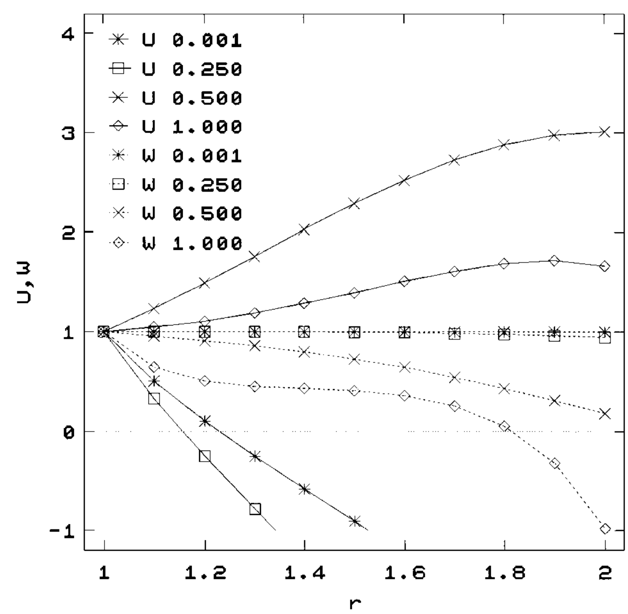

Fig. 2 Normalized amplitudes of displacements for the basic curve.

for some K'. Note the presence of the nodal circles for $K' = 0.001$ and 0.250 (at $r = 1.15$ and 1.23) for the axial displacements and for $K' = 1.000$ (at $r = 1.81$) for the radial displacements.

Conclusions

The first five dispersion curves were determined and shown for the axisymmetric elastic waves propagating along the axis of an infinitely long cylindrical pipe of orthotropic boron–epoxy. Boron fibers coincide with the circumferential coordinate of the cylindrical coordinate system. The coupled partial differential equations of motion with variable coefficients were reduced to ordinary differential equations with constant coefficients. A convergence test of the method was done for chosen wave numbers. After small corrections, the method is advisable for the arbitrarily laminated cylinders with various kinds of anisotropy of the material layers.

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